Amplification by Interdigital Excitation of Space-Charge Waves in Semiconductors

HENRI BAUDRAND, TANOS EL KHOURY, AND DÉSIRÉ LILONGA

Abstract —**A new concept of amplification of the electromagnetic (EM) wave as a consequence of its interaction with a space-charge wave in a semiconductor is analyzed. The EM wave is applied to an interdigital line which in turn excites a space-charge wave in a high-resistivity silicon. The theoretical calculations are carried out by means of the least-square boundary residual method, where a theoretical gain of 84 dB is obtained at synchronism of the third harmonic of the wave. The experimental device exhibits a net gain of 13 dB at synchronism. The mobility of the carriers in the semiconductor is deduced out of the experimental results.**

I. Introduction

THE CREATION of space-charge waves in semiconductors is the basis of the amplification phenomenon in many amplifying devices. This wave, which is produced by the charge modulation, should be drifted through the semiconductor by the application of a dc voltage. The charge modulation can be used either directly, as in transit-time devices like IMPATT diodes which offer gain when the half-period of the input signal is approximately equal to the transit-time [1], [2] or by achieving its coupling with a slow wave. In the latter case, the interaction occurs when the velocity of the space-charge wave is greater than or equal to that of the slow wave, as in acoustic [3], magnetostatic [4], or electromagnetic slow-wave devices. As far as the electromagnetic wave is concerned, the idea of perfecting the solid-state version of the traveling-wave tube has encouraged many authors, among whom we mention just a few pioneers, to investigate the possibilities of obtaining the coupling between the slowed wave and the drift current of carriers in a semiconductor [5]–[11]. The fact that the drift velocity in a semiconductor exceeds hardly the limit of 10^5 m/s requires that the slow-wave structure, such as the meander or interdigital lines, offer a delay rate of about 1000 in order to achieve a good coupling between the wave and the carriers. In these conditions, losses and dimensional problems are serious limitations which cannot be overcome easily.

It does not seem necessary that the electromagnetic wave has a group velocity synchronized with that of the carriers, but the coupling of the carriers with space-harmonics of the EM field can provide gain, as in superlattices in which the semiconductor resistivity is modulated periodically [12].

In this paper, we propose a new type of structure for which we study the interaction between the space-charge wave produced in a high-resistivity semiconductor and the space harmonics of the field produced by an interdigital line.

The role of this line differs from that of a slow-wave structure, as the length of the conductors is not enough to permit the propagation of the wave along the line fingers and so losses are considerably reduced. The mechanism of operation consists in producing a modulation of the charge in the semiconductor by the application of the ac voltage to the input, and then drifting the modulated charge along the semiconductor with the aid of a dc voltage, hence producing a space-charge wave. Now any variation in the charge density within the semiconductor would modify the charge distribution on the conductors of the line by electrostatic influence. We may then expect an efficient interaction when the wavelength of the space-charge wave is equal to the period of the interdigital line, or to whole fractions of the period for higher-order space-harmonics. In order to reduce the attenuation of the space-charge wave, the semiconductor should be a high-resistivity one.

II. Theory

The structure under study is shown in Fig. 1. The interdigital line provides the periodicity of the device but, unlike a classical delay line, the small length of its fingers does not permit a propagation phenomenon along the conductors at the frequencies used. The input and output circuits are coupled capacitively to the line so that each finger can be electrically isolated from the rest of the structure. To develop the physical model, we make the following assumptions:

The semiconductor is an n-type one, where only electrons are considered as carriers.

The electron dynamics are adequately described by the classical Boltzmann transport equation with dominant collision assumption.

No magnetic field is applied to the system, the temperature gradient is neglected and the local electrical field is described by Poisson's equation.

For a small signal analysis, assuming sinusoidal time variations, the following fundamental equations can be

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H. Baudrand and D. Lilonga are with the Laboratoire de Microondes E.N.S.E.E.I.H.T., 31071 Toulouse, France.

T. El Khoury is with the Department de Physique, Faculte de Sciences II, University of Lebenon, Beruit, Lebenanon.

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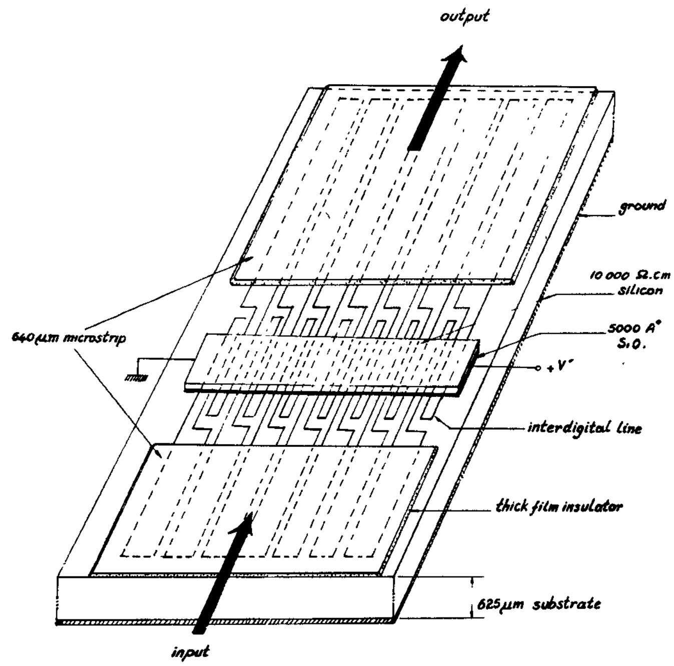

Fig. 1. The amplifier structure.

written in the semiconductor:

$$\nabla \times \vec{H} = \vec{J} + j\omega\epsilon_s\vec{E} \tag{1}$$

$$\nabla \times \vec{E} = -j\omega\mu_0\vec{H} \tag{2}$$

$$\nabla \cdot \vec{B} = 0 \tag{3}$$

$$\nabla \cdot \vec{J} = -j\omega\rho \tag{4}$$

$$\vec{J} = \sigma_0\left(\vec{E} - \frac{\lambda_D^2}{\epsilon_s}\nabla\rho\right) + \rho\vec{v}_0 \tag{5}$$

where $\lambda_D = [\epsilon_s U_T/\rho_0]^{1/2}$ is the Debye length, σ_0 and ϵ_s are the conductivity and dielectric constant of the semiconductor, ρ_0 is the charge density at equilibrium, v_0 the drift velocity in $\overline{oy}$ direction (Fig. 2), and $U_T = kT/q$ is the thermal potential. Combining (4) and (5), we obtain the propagation equation of the charge density ρ in the semiconductor

$$\nabla^2\rho = \left(1 + j\omega\frac{\epsilon_s}{\sigma_0}\right)\frac{\rho}{\lambda_D^2} + \frac{\epsilon_s\vec{v}_0\cdot\nabla\rho}{\sigma_0\lambda_D^2}. \tag{6}$$

The structure periodicity leads to find a solution of (6) as series expansion of space harmonics

$$\rho = \sum_{n=-\infty}^{+\infty} \rho_n \exp\left[\alpha_n' x + jn\beta y\right] \tag{7}$$

where

$$\alpha_n' = \pm\left[\frac{\rho_0}{\epsilon_s U_T} + \beta^2 n^2 + j\frac{\rho_0}{\sigma_0 U_T}(\omega + n\beta v_0)\right]^{1/2}$$

and $\beta = 2\pi/p$ with $p =$ period of the structure. We note that the real part of α_n' must be nonnegative, as the charge expansion is considered only in the semiconductor (Fig. 2).

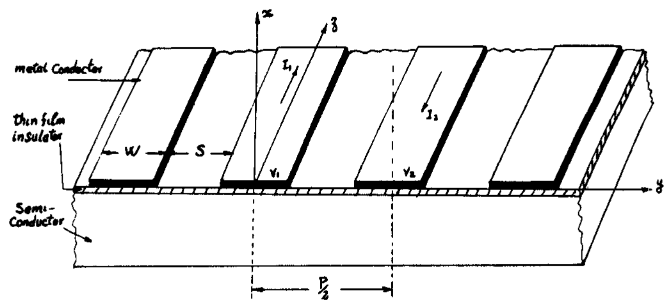

Fig. 2. Unit cell of the periodic structure.

Quasi-Static Assumption

The complete study of space-charge waves in a semiconductor shows the existence of TE and TM modes [5], [8], but we will consider only TM modes, as the longitudinal component of the electric field Ey should be nonzero. At the frequencies used, the quasi-static assumption is valid so that the electrical potential satisfies the following equations inside and outside the semiconductor:

$$\begin{cases} \nabla^2 V = -\dfrac{\rho}{\epsilon_s}, & x \leqslant 0 \\ \nabla^2 V = 0, & x > 0 \end{cases} \tag{8}$$

solutions of (8) are also expansions of space harmonics so that

$$V(x,y) = \begin{cases} \sum_n (A_n \exp[-\alpha_n x + jn\beta y], & x > 0 \\ \sum_n \left(B_n \exp[\alpha_n x] + C_n \exp[\alpha_n' x]\right)\exp[j_n\beta y], & x \leqslant 0 \end{cases} \tag{9}$$

where $\alpha_n = |n\beta|$ and A_n, B_n, and C_n are amplitude constants to be determined by the boundary conditions:

i) Ey is continuous at

$$x = 0 \rightarrow A_n = B_n + C_n. \tag{10}$$

ii) The normal component of the total current density is zero at the insulator–semiconductor interface (the insulator is supposed to be very thin)

$$J_x|_{x=0} = \sigma_0\left[E_x - \frac{\lambda_D^2}{\epsilon_s}\cdot\frac{\partial\rho}{\partial x}\right]_{x=0} = 0 \Rightarrow B_n = b_n C_n \tag{11}$$

where

$$b_n = \frac{\alpha_n'}{\alpha_n}\left[1 + \lambda_D^2\left(\alpha_n^2 - \alpha_n'^2\right)\right], \qquad \text{with } \rho_n = \epsilon_s C_n\left(\alpha_n^2 - \alpha_n'^2\right)$$

using the relations (10) and (11), the electric field Ey and the surface charge density ρ_s can be written as

$$Ey = \sum_n -jX_n \exp[jn\beta y] \tag{12}$$

$$\rho_s = \epsilon_0 \sum_n D_n X_n \exp[jn\beta y] \tag{13}$$

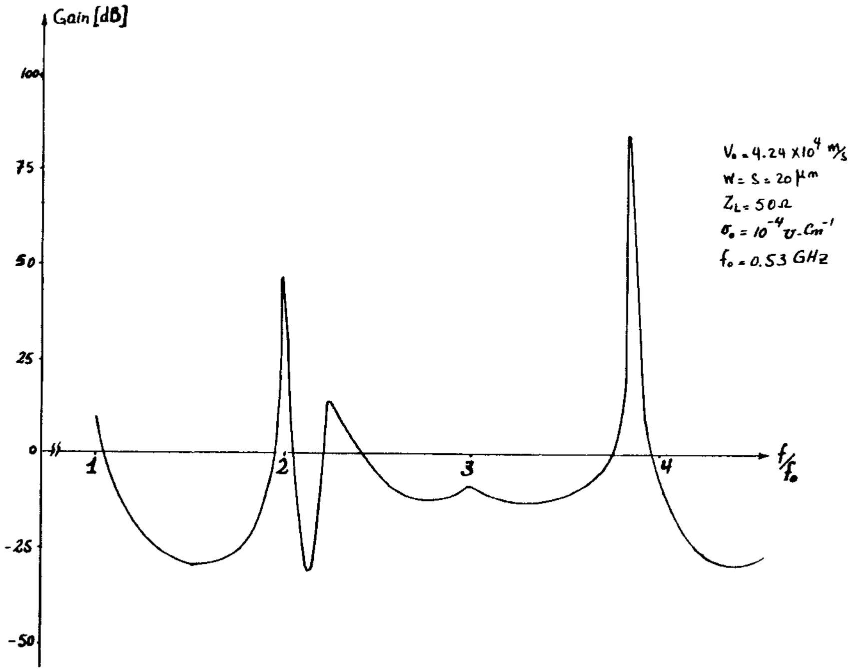

Fig. 3. Theoretical gain.

where

$$X_n = n\beta A_n$$

and

$$D_n = \frac{|n|}{n}\left[1 + \frac{\epsilon_s}{\epsilon_0 A_n}\left(B_n + \frac{\alpha_n'}{\alpha_n} C_n\right)\right].$$

Now, the problem is reduced to the determination of the only unknown series, i.e., X_n parameters. To do this, we notice that Ey and ρ_s satisfy the following boundary conditions:

$$Ey(0, y) = -j\left[\sum_n X_n \exp\left[jn\beta y\right]\right] = 0$$

on the conductor surface (14)

$$\rho_s(0, y) = \epsilon_0\left[\sum_n D_n X_n \exp\left[jn\beta y\right]\right] = 0$$

between the conductors. (15)

The couple of (14) and (15) should be solved simultaneously for X_n. The calculations are carried out by the least-square boundary residual method (Appendix I) where the total charge on the conductors and the potential distribution can be deduced immediately. Currents and voltages of the conductors are found to be

$$\begin{aligned} I &= j\omega Q \\ &= j\omega \int_{\text{Cond.}} L\rho_s \, dy \\ &= j\omega\epsilon_0 L \int_{\text{Cond.}} \left[\sum_n D_n X_n \exp\left[jn\beta y\right]\right] dy \qquad (16) \end{aligned}$$

$$V(0, y) = \frac{1}{\beta}\sum_n \frac{X_n}{n} \exp\left(jn\beta y\right) \qquad (17)$$

where L is the finger length in the z direction. In order to determine the total gain, we consider the structure as a serial association of two-port networks, each representing the unit cell of the structure which embraces two conductors (Fig. 2). Using (16) and (17), currents and voltages can be calculated on the two conductors of the unit cell, yielding the characteristics of the elementary network and consequently those of the overall network (Appendix II). The potential on the conductors should be practically constant, but in our case, we have chosen the potential values at $y = (W/4)$ and $y = (p/2) + (W/4)$ which constitute the best evaluation of the voltages of the two conductors of the unit cell. Figs. 3 and 4 show the variations of gain and stability factor for the device specifications indicated. The theoretical study predicts maximum gain for the second and fourth harmonics of the fundamental frequency, where the stability factor has minima for the odd harmonics so that there would be possibility of obtaining gain for all of the harmonics. The experiment will confirm this fact, as we will see later. The ac voltage applied to the half of the conductors of the line, creates different potentials on the consecutive fingers setting up an electric field which reacts with the drift electrons in the semiconductors by proximity effect.

Reciprocally, any change in the charge density within the semiconductor results in variations of the charge distribution on the conductors of the line by electrostatic effect. The efficient interaction between the field and the space-charge wave occurs when the wavelength of the latter is equal to the period of the interdigital line p (or whole fractions of p for higher order space-harmonics). The space-charge wave length is

$$\lambda_{sc} = \frac{v_0}{f} \qquad (18)$$

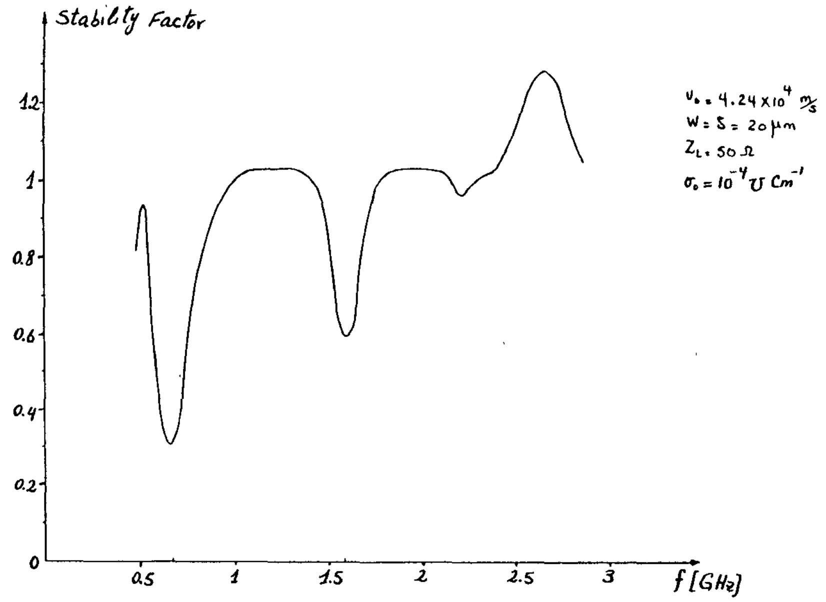

Fig. 4. Stability factor variations with the frequency.

where f is the signal frequency. Therefore, the synchronism condition would imply

$$\lambda_{sc} = \frac{p}{n} \tag{19}$$

where n is the order of the harmonics of the EM field. As long as the mobility can be assumed constant in the bias range, we can write

$$\frac{V_1'}{f_1} = \frac{V_2'}{f_2} = \frac{p}{n} \tag{20}$$

where V_1' and V_2' are two different bias voltages and f_1, f_2 are the corresponding frequencies for which synchronism occurs with the same space-harmonic. The experimental results verify this proportionality relation. It should be noted that in order to carry out a rigorous analysis of the problem, a spatial modulation of the charge in the semiconductor should be taken into account across the width of the conductor i.e; in the drift direction. In fact there would be opposite space-charge layers at the ends of the conductor width [13]. However, the semiconductor used here is a high-resistivity one and it acts more as a dielectric than a conductor at the frequencies used, so that these kind of field effects can be neglected without introducing a significant error.

III. Experimental Results

The structure used for the experimental study is that of Fig. 1. The interdigital line is composed of eight pairs of gold conductors on alumina substrate, where the width and the spacing of the fingers is 20 μm. With this choice, the total width of the structure would correspond to input and output impedances of 50 Ω for the feeding and outgoing microstrips. The length of conductors is 1 mm, not enough to permit a propagation phenomenon at the range of frequencies used (1–2 GHz). In order to avoid short-circuits, each conductor is isolated from the neighboring ones and the input and output circuits are capacitively coupled to the line through an insulating film deposited on the line.

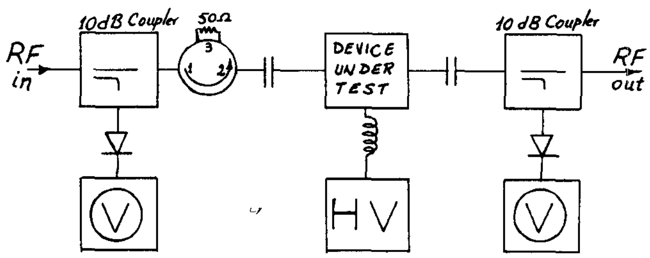

Fig. 5. Schematic diagram of the experimental set up.

A silicon bar 3 mm long, 0.9 mm large, and 30 μm thick has been laid on the line, isolated from it by a thin film of 0.5 μm thickness. The bias voltage is applied to the semiconductor through ohmic contacts 1 mm apart, where the high resistivity of the silicon ($\sigma_0 = 10^{-4}$ Ω cm^{-1}) is supposed to reduce the propagation losses of the space-charge wave. The semiconductor can be made thinner if it is more heavily doped. In order to measure the voltage gain, the experimental set-up of Fig. 5 has been used. In a first step, the transfer characteristics of the device with no bias voltage to the semiconductor has been measured and the results are illustrated on Fig. 6, where only one important resonance is observed for $f = 0.9$ GHz. Then, experimental data have been obtained for two different bias voltages of 400 and 450 V for $V' = 450$ V, there maxima show up (Fig. 7). The maximum gain of 13 db corresponds to the third harmonic of the fundamental frequency. It can be noted that the maxima occur for the frequencies of 1.07 GHz, 1.6 GHz, and 2.12 GHz, respectively, with a difference of approximately 0.53 GHz between the consecutive ones.

The synchronism relation is

$$\frac{v_0}{f} = \frac{p^0}{n} \tag{21}$$

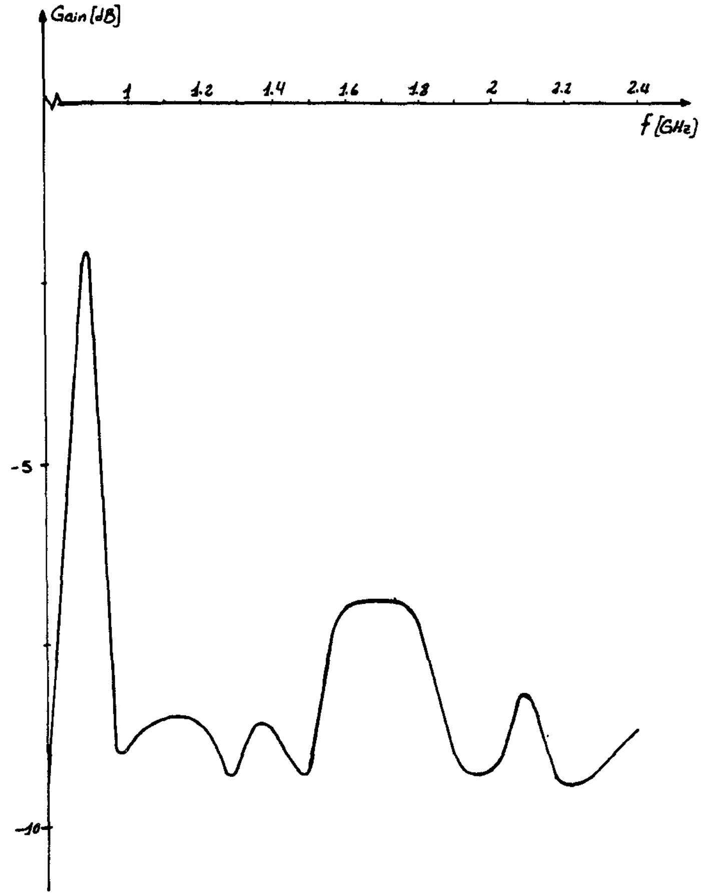

Fig. 6. Transfer characteristics of the passive device.

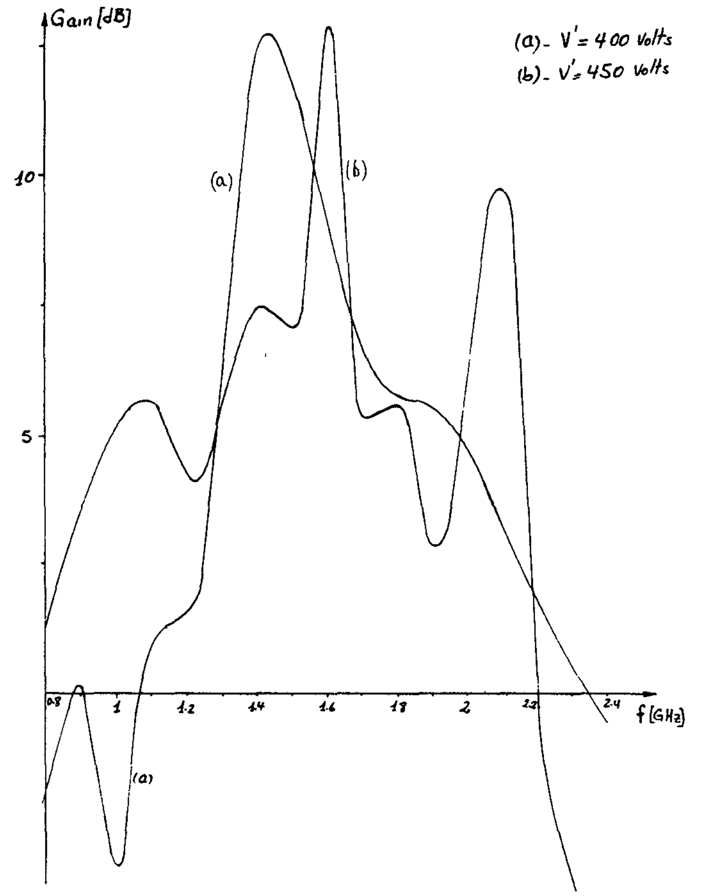

Fig. 7. Experimental gain versus frequency.

giving the values of 2, 3, and 4 for n, where the fact that the second, third, and fourth harmonics are synchronized with the space-charge wave is confirmed. With $p = 80$ μm, we find a drift velocity of $v_0 = 4.24 \times 10^4$ m/s for the carriers. The electric field in the semiconductor is $E_0 = 450 \times 10^3$ V/m and the mobility of the electrons is found to be $\mu_n = 924$ cm²/V·s. The mobility of the same sample of silicon was measured in another experiment by an acoustic interaction technique, with a drift velocity of 3.48×10^3 m/s and the obtained value was 1268 cm²/V·s. This difference may be interpreted in two ways: either the mobility decreases as saturation is approached, or the surface state of the silicon play a major role. The experiment reveals that an important gain occurs for the third harmonic, which is not obtained directly in the theoretical gain curve. However, as explained before, the difference is due to a net decrease of the stability factor at this frequency.

IV. Conclusion

A new concept of the excitation of a space-charge wave in a semiconductor by an interdigital line has been discussed and an original method is proposed to analyze the physical problem. The theoretical possibilities of interaction between space-harmonics of an EM field produced by an interdigital line and the space-charge wave in a semiconductor have been verified experimentally. The comparison between the experimental data obtained for the biased and unbiased semiconductor proves that the device can provide net gain and the occurrence of output peaks is consistent with the theory. The gain obtained experimentally is less than the expected one, since losses have not been taken into account in the theory and no matching element has been used in the experimental setup. In order to increase the gain, it would be possible to extend the interaction path by increasing the number of conductors of the line, where the interaction would take place at lower frequencies.

The generation and amplification of space-charge waves as done in this experiment allows their use in signal processing as done with acoustic or magnetostatic waves. The use of semiconductor as substrate permits the elaboration of the device in a fully integrated technology with a frequency rise bonus. With a structure period of 8 μm, we can expect an excitation at 20 GHz. Moreover, if the surface of the semiconductor is perfectly controlled, we may hope to build 80-GHz amplifiers in planar technology with a conductor width of 0.5 μm. At last, the least-square boundary residual method seems quite well adapted to this kind of analysis, as the time savings in the numerical treatment of the problem is not negligible.

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Appendix I
Least-Square Boundary Residual Technique

The least-square residual method is an approximation method which is applied in order to obtain the best solution of the problems characterized by N conditions satisfied by N_0 variables [14], [15].

Let the domain D be defined, when the electromagnetic field can be expressed in a ψ_n basis, truncated up to the order N_0.

For each point M_i of D, the Maxwell equations can be written as

$$\sum_{n=0}^{N_0} a_n(i) \cdot X_n = 0, \qquad \text{with } i = 1, 2, \cdots, N. \tag{A1}$$

In general, the equations are not valid simultaneously due to this truncature. The least-square technique consists in finding the best solution by the minimization of the following expression:

$$f(X1, X2, \cdots, X_n) = \sum_{i=1}^{N} \left| \sum_{n=0}^{N_0} a_n(i) \cdot X_n \right|^2 \tag{A2}$$

where the expression $\sum_{n=0}^{N_0} a_n(i) \cdot X_n$ is called the residue at point M_i.

Now, if we introduce a variable y with continuous variation, we will have

$$a_n(i) = a_n(y_i)$$

and we obtain

$$f(X_1, X_2, \cdots, X_n) = \int_{y \in D} \left| \sum_{n=0}^{N_0} a_n(y) \cdot X_n \right|^2 dy \tag{A3}$$

and the expression to be minimized:

$$f(x) = \int_{y \in D} \left[\sum_{m=0}^{N_0} \sum_{n=0}^{N_0} a_m^*(y) a_n(y) X_m^* X_n \right] dy \tag{A4}$$

$$f(x) = \sum_{m,n} A_{m,n} X_m^* X_n \qquad \text{with } X = (X_1, X_2, \cdots, X_n) \tag{A5}$$

and

$$A_{m,n} = \int_{y \in D} a_m^*(y) \cdot a_n(y)\, dy. \tag{A6}$$

This $A_{m,n}$ matrix is called the least-square matrix, as it is the representation of an operator A in the $\{\psi_n\}$ basis

$$A_{m,n} = \langle \psi_m | \hat{A} | \psi_n \rangle = \langle a_m(y) | a_n(y) \rangle. \tag{A7}$$

We have to solve the equation

$$\langle X | A | X \rangle = \lambda \langle X | X \rangle. \tag{A8}$$

Here, the minimum satisfies the equation

$$AX = \lambda X. \tag{A9}$$

Among all of the eigenvalues λ, we will look for the smallest one, because at the minimum we have

$$\langle X | A | X \rangle = \lambda_{\min} \langle X | X \rangle. \tag{A10}$$

It can be noted that, in the case of a periodic structure, the integration on the D domain is reduced to an integration on one period. One notes that, for the interdigital line, we have taken

$$a_n(y) = \begin{cases} \exp[jn\beta y] & \text{on the conductors} \\ D_n \exp[jn\beta y] & \text{between the conductors.} \end{cases} \tag{A11}$$

Appendix II
Typical Characteristics of the Equivalent Network

The whole structure studied in the paper may be considered as a serial association of elementary cells, each one including two consecutive conductors, but as for their electrical behavior, these cells are connected in parallel. Our approach consists of representing each unit cell as a two-port network, where the two conductors are the input and output ports, and we have

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [Z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{A12}$$

where $[Z]$ is the impedance matrix of the unit cell. The small signal assumption, hence the linear behavior of the network, allows the application of the superposition principle. As the unit cell includes two conductors, it seems reasonable to superpose the two eigenmodes associated to the two smallest eigenvalues of the least-square matrix (Appendix I). If X and X' are taken as the two eigenvectors corresponding to these two smallest eigenvalues, the (A12) becomes

$$\begin{bmatrix} V_{1X} \\ V_{2X} \end{bmatrix} = [Z] \begin{bmatrix} I_1 X \\ I_2 X \end{bmatrix} \tag{A13}$$

and

$$\begin{bmatrix} V_{1X'} \\ V_{2X'} \end{bmatrix} = [Z] \begin{bmatrix} I_{1X'} \\ I_{2X'} \end{bmatrix} \tag{A14}$$

and the elements of the impedance matrix will be

$$Z_{11} = \frac{V_{1X} I_{2X'} - I_{2X} V_{1X'}}{\Delta} \tag{A15}$$

$$Z_{12} = \frac{I_{1X} V_{1X'} - V_{1X} I_{1X'}}{\Delta} \tag{A16}$$

$$Z_{21} = \frac{V_{2X} I_{2X'} - I_{2X} V_{2X'}}{\Delta} \tag{A17}$$

$$Z_{22} = \frac{I_{1X} V_{2X'} - I_{1X'} V_{2X}}{\Delta} \tag{A18}$$

where

$$\Delta = I_{1X} I_{2X'} - I_{2X} I_{1X'}.$$

In order to obtain the matrix impedance of the whole line $[Z']$, the elements of the $[Z]$ are divided by N, where N is the number of elementary cells, connected in parallel, to constitute the interdigital line.

The stability factor, directivity, and the power gain may be readily deduced from the $[Z']$ matrix [16]–[18].

A. Stability Factor

The degree of stability of the two-port network can be described by the Linvill's parameter

$$k = \frac{2\mathscr{R}(\gamma_{11}) \cdot \mathscr{R}(\gamma_{22}) - \mathscr{R}(\gamma_{12} \cdot \gamma_{21})}{|\gamma_{21} \cdot \gamma_{12}|} \tag{A19}$$

which is invariant under immittance substitution

$$\gamma \to Z, Y, h, \text{and } g.$$

The criterion for unconditional stability may be written as $k \geqslant 1$, provided $\mathscr{R}(Z'_{11})$ and $\mathscr{R}(Z'_{22}) > 0$. When $-1 \leqslant k < 1$, the network is in the region of conditional stability.

B. Directivity

The nonreciprocity of the network is measured by the ratio

$$d = \frac{\gamma_{21}}{\gamma_{12}} \tag{A20}$$

which is also invariant under the immittance substitution.

C. Power Gain

If Z_L is the value of the load impedance, power gain is defined as

$$G = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{\mathscr{R}[Z_L]}{\mathscr{R}\left\{Z_{\text{in}} \left| \frac{Z_L + Z'_{22}}{Z'_{21}} \right|^2 \right\}} \tag{A21}$$

where Z_{in} is the input impedance.

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Henri Baudrand was born in 1939. He obtained the "Diplôme d'Ingénieur" in electronics and the Docteur-es-Sciences degree in microwaves, both from E.N.S.E.E.I.H.T., Toulouse, France, in 1962 and 1966, respectively.

Since then, he has been working on active and passive microwave integrated circuits in the "Laboratoire de Microondes" of E.N.S.E.E.I.H.T. He is a Professor of Physics in this school.

Tanos El Khoury photograph and biography not available at the time of publication.

Désiré Lilonga was born in Ekouli, Congo, in 1956. He obtained the "Maîtrise de Physique" from the Pau University in 1980 and the "Diplôme d'Etudes Approfondies" in physics from the Bordeaux University in 1981.

He then joined Laboratoire de Microonde in Toulouse, France, where he has been working on the coupling between the space-charge wave in semiconductors and an interdigital structure.

The Electrostatic Field of Conducting Bodies in Multiple Dielectric Media

SADASIVA M. RAO, MEMBER, IEEE, TAPAN K. SARKAR, SENIOR MEMBER, IEEE, AND ROGER F. HARRINGTON, FELLOW, IEEE

Abstract—**A method for computing the electrostatic fields and the capacitance matrix for a multiconductor system in a multiple dielectric region is presented. The number of conductors and the number of dielectrics in this analysis are arbitrary. Some of the conductors may be of finite volume and others may be infinitesimally thin. The conductors can be either above a single ground plane or between two parallel ground planes. The formulation is obtained by using a free-space Green's function in conjunction with total charge on the conductor-to-dielectric interfaces and polarization charge on the dielectric-to-dielectric interfaces. The solution is effected by the method of moments using triangular subdomains with piecewise constant expansion functions and point matching for testing. Computed results are given for some finite-length conducting lines, compared to previous results obtained by two-dimensional analysis.**

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S. M. Rao and T. K. Sarkar are with the Department of Electrical Engineering, Rochester Institute of Technology, Rochester, NY 14623.

R. F. Harrington is with the Department of Electrical Engineering, Syracuse University, Syracuse, NY 13210.

I. Introduction

THE OBJECTIVE of this paper is to compute the electrostatic fields and the capacitance matrix of arbitrarily shaped conductors embedded in multiple dielectric regions. The entire system could be situated over a finite or infinite ground plane, or could be between two ground planes. This solution is useful for finding equivalent circuits of microstrip junctions and discontinuities and for vias connecting conductors located in various dielectric regions. Some of the conductors may be of finite volume and others may be infinitesimally thin.

Recent advances in integrated circuit technology, such as VLSI design in the microwave region, necessitate a sophisticated analysis, design, and construction of transmission lines to carry signals from one end to the other. Even though a large volume of literature exists to analyze an infinitely long transmission line, there are very few satisfactory procedures to solve for the equivalent circuits

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